Lesson Summary

**THEOREM:** Given a dilation with center $O$ and scale factor $r$, then for any two points $P$ and $Q$ in the plane so that $O$, $P$, and $Q$ are not collinear, the lines $PQ$ and $P'Q'$ are parallel, where $P' = \text{Dilation}(P)$ and $Q' = \text{Dilation}(Q)$, and furthermore, $|P'Q'| = r|PQ|$.

Problem Set

1. Use a piece of notebook paper to verify the fundamental theorem of similarity for a scale factor $r$ that is $0 < r < 1$.
   - Mark a point $O$ on the first line of notebook paper.
   - Mark the point $P$ on a line several lines down from the center $O$. Draw a ray, $\overrightarrow{OP}$. Mark the point $P'$ on the ray and on a line of the notebook paper closer to $O$ than you placed point $P$. This ensures that you have a scale factor that is $0 < r < 1$. Write your scale factor at the top of the notebook paper.
   - Draw another ray, $\overrightarrow{OQ}$, and mark the points $Q$ and $Q'$ according to your scale factor.
   - Connect points $P$ and $Q$. Then, connect points $P'$ and $Q'$.
   - Place a point, $A$, on the line containing segment $PQ$ between points $P$ and $Q$. Draw ray $\overrightarrow{OA}$. Mark point $A'$ at the intersection of the line containing segment $P'Q'$ and ray $\overrightarrow{OA}$.

   a. Are the lines containing segments $PQ$ and $P'Q'$ parallel lines? How do you know?
   b. Which, if any, of the following pairs of angles are equal in measure? Explain.
      i. $\angle OPQ$ and $\angle OP'Q'$
      ii. $\angle OAP$ and $\angle O'A'P'$
      iii. $\angle OAP$ and $\angle O'A'Q'$$
      iv. $\angle OQP$ and $\angle O'Q'P'$
   c. Which, if any, of the following statements are true? Show your work to verify or dispute each statement.
      i. $|OP'| = r|OP|$
      ii. $|OQ'| = r|OQ|$
      iii. $|P'A'| = r|PA|$
      iv. $|A'Q'| = r|AQ|$
   d. Do you believe that the fundamental theorem of similarity (FTS) is true even when the scale factor is $0 < r < 1$? Explain.
2. Caleb sketched the following diagram on graph paper. He dilated points $B$ and $C$ from center $O$.

\[
\begin{array}{c}
\text{O} \\
\text{B'} \\
\text{C'} \\
\text{C} \\
\text{B}
\end{array}
\]

a. What is the scale factor $r$? Show your work.
b. Verify the scale factor with a different set of segments.
c. Which segments are parallel? How do you know?
d. Which angles are equal in measure? How do you know?

3. Points $B$ and $C$ were dilated from center $O$.

\[
\begin{array}{c}
\text{O} \\
\text{B'} \\
\text{C'} \\
\text{C} \\
\text{B}
\end{array}
\]

a. What is the scale factor $r$? Show your work.
b. If $|OB| = 5$; what is $|OB'|$?
c. How does the perimeter of triangle $OBC$ compare to the perimeter of triangle $OB'C'$?
d. Did the perimeter of triangle $OB'C' = r \times \text{perimeter of triangle } OBC$? Explain.