Lesson 9: Sequencing Rotations

Classwork

Exploratory Challenge

1. a. Rotate \( \triangle ABC \) \( d \) degrees around center \( D \). Label the rotated image as \( \triangle A'B'C' \).
   
b. Rotate \( \triangle A'B'C' \) \( d \) degrees around center \( E \). Label the rotated image as \( \triangle A''B''C'' \).
   
c. Measure and label the angles and side lengths of \( \triangle ABC \). How do they compare with the images \( \triangle A'B'C' \) and \( \triangle A''B''C'' \)?
   
d. How can you explain what you observed in part (c)? What statement can you make about properties of sequences of rotations as they relate to a single rotation?
2.

a. Rotate $\triangle ABC \ d$ degrees around center $D$, and then rotate again $d$ degrees around center $E$. Label the image as $\triangle A'B'C'$ after you have completed both rotations.

b. Can a single rotation around center $D$ map $\triangle A'B'C'$ onto $\triangle ABC$?

c. Can a single rotation around center $E$ map $\triangle A'B'C'$ onto $\triangle ABC$?

d. Can you find a center that would map $\triangle A'B'C'$ onto $\triangle ABC$ in one rotation? If so, label the center $F$. 
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3.

a. Rotate \( \triangle ABC \) 90° (counterclockwise) around center \( D \), and then rotate the image another 90° (counterclockwise) around center \( E \). Label the image \( \triangle A'B'C' \).

b. Rotate \( \triangle ABC \) 90° (counterclockwise) around center \( E \), and then rotate the image another 90° (counterclockwise) around center \( D \). Label the image \( \triangle A''B''C'' \).

c. What do you notice about the locations of \( \triangle A'B'C' \) and \( \triangle A''B''C'' \)? Does the order in which you rotate a figure around different centers have an impact on the final location of the figure’s image?
4. a. Rotate $\triangle ABC$ $90^\circ$ (counterclockwise) around center $D$, and then rotate the image another $45^\circ$ (counterclockwise) around center $D$. Label the image $\triangle A'B'C'$.

b. Rotate $\triangle ABC$ $45^\circ$ (counterclockwise) around center $D$, and then rotate the image another $90^\circ$ (counterclockwise) around center $D$. Label the image $\triangle A''B''C''$.

c. What do you notice about the locations of $\triangle A'B'C'$ and $\triangle A''B''C''$? Does the order in which you rotate a figure around the same center have an impact on the final location of the figure’s image?
5. \( \triangle ABC \) has been rotated around two different centers, and its image is \( \triangle A'B'C' \). Describe a sequence of rigid motions that would map \( \triangle ABC \) onto \( \triangle A'B'C' \).