Lesson 5: Negative Exponents and the Laws of Exponents

Classwork

**Definition:** For any nonzero number $x$, and for any positive integer $n$, we define $x^{-n}$ as $\frac{1}{x^n}$.

Note that this definition of negative exponents says $x^{-1}$ is just the reciprocal, $\frac{1}{x}$, of $x$.

As a consequence of the definition, for a nonnegative $x$ and all integers $b$, we get

$$x^{-b} = \frac{1}{x^b}$$

**Exercise 1**

Verify the general statement $x^{-b} = \frac{1}{x^b}$ for $x = 3$ and $b = -5$.

**Exercise 2**

What is the value of $(3 \times 10^{-2})$?
Exercise 3
What is the value of \((3 \times 10^{-5})\)?

Exercise 4
Write the complete expanded form of the decimal 4.728 in exponential notation.

For Exercises 5–10, write an equivalent expression, in exponential notation, to the one given, and simplify as much as possible.

Exercise 5
\[5^{-3} = \]

Exercise 6
\[\frac{1}{8^3} = \]

Exercise 7
\[3 \cdot 2^{-4} = \]

Exercise 8
Let \(x\) be a nonzero number.
\[x^{-3} = \]

Exercise 9
Let \(x\) be a nonzero number.
\[\frac{1}{x^3} = \]

Exercise 10
Let \(x, y\) be two nonzero numbers.
\[xy^{-4} = \]
We accept that for nonzero numbers $x$ and $y$ and all integers $a$ and $b$,
\[ x^a \cdot x^b = x^{a+b} \]
\[ (x^b)^a = x^{ab} \]
\[ (xy)^a = x^a y^a. \]

We claim
\[ \frac{x^a}{x^b} = x^{a-b} \quad \text{for all integers } a, b. \]
\[ \left( \frac{x}{y} \right)^a = \frac{x^a}{y^a} \quad \text{for any integer } a. \]

**Exercise 11**
\[ \frac{19^2}{19^5} = \]

**Exercise 12**
\[ \frac{17^{1.6}}{17^{-3}} = \]

**Exercise 13**
If we let $b = -1$ in (11), $a$ be any integer, and $y$ be any nonzero number, what do we get?

**Exercise 14**
Show directly that \[ \left( \frac{7}{5} \right)^{-4} = \frac{7^{-4}}{5^4}. \]